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Level-set based method for designing novel brushed synchronous machines

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SUMMARY

Nowadays, research on electromagnetic devices increasingly focuses on multiphysics three dimensional complex systems. However, this kind of simulations require huge computational resources and consequently very high CPU time. In [1] the authors proposed a level-set based method allowing meshing step savings in any iterative processes (as movement modeling or optimization processes). The level-set method lies in implicit description of moving fronts [2]. Inspired by [3], all physical parameters (material properties, supply or armature movement) are implemented by projection of mathematical functions. In this paper, the authors proposes to study and design a new kind of mechanical commutator for DC machines thanks to this coupled method. Indeed, there is a renewed interest in DC machines because they can be suitable for small mobilities. To be noted, small mobility concerns all kind of vehicles (2 wheels - 4 wheels - autonomous vehicles such as drones) that have speeds below 50 km/h. However, specifications for traction applications are difficult to achieve for such a rustic machine and requires some modifications. That is why, inspired by usual synchronous machine, the commutator has been redesigned to obtain expected theoretically signal waveform. Furthermore, numerical coupled simulations have been used to design both machine and supply simultaneously. This work has led to the manufacture of a proof of concept and preliminary results are very promising. Copyright © 202X John Wiley & Sons, Ltd.

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KEY WORDS: DC machines, Coupled simulation, FEM, Multiphysics

1. INTRODUCTION

The authors focus on the modeling and optimization of commutator machines used in powertrains of small electric vehicles. This kind of machines represents a credible alternative because of their competitiveness, robustness and reliability mainly due to the lack of power electronics, although the presence of the mechanical switching system also brings its own disadvantages (sparking, mechanical wear). Nevertheless, to take into account commutator power supply, it is necessary to implement a particular numerical magnetic model in finite element modeling. In a previous article, the authors developed an original method that performs projection of physical properties and sources which avoids a remeshing, and applied it to the design problem of a permanent magnet synchronous machine. In the present paper, they extend the concept to efficiently model the commutator system of brushed DC machines. Indeed, this method can lead to the coupling of the magnetic problem and the external electrical circuit to the current flow formulation at the commutator interface which make it easy to model brushed DC machine as only few methods exist as detailed in [4]. Then, in

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this article, we extend this technique to an innovative brushed DC machine with a synchronous type of power supply, in other words: a synchronous brushed machine.

2. INTEREST OF THE COUPLED APPROACH FOR DC MACHINE COMMUTATOR MODELING

A crucial issue in DC machine modeling is the commutator [5, 6, 7]. DC commutator is a very low-tech system which allows for creating alternative signals and insure current distribution in windings. System shown in Figure 1 presents a brush made of carbon and several segments of copper. This apparent simplicity does not represent the complexity of the phenomena involved. This commutator belongs to a 1.2 kW automotive starter. In the following, we will see that projection method is suitable to build FE-model for commutator that can be used instead of usual coupled circuit models.

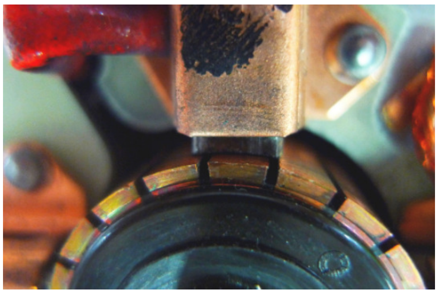

Figure 1. Mechanical commutator of a DC machine.

2.1. Standard component approach

Usually dynamic magnetic simulations require the implementation of a circuit model. This circuit part describes the windings and the power supply. Then, a coupling between the magnetic model and the electric circuit allows to take into account the inductive effect of the windings. In the case of brushed DC machines, the most common way to implement the commutator system consists in including the latter in the electric circuit [8]. Indeed, with an electronic point of view, the commutator can be easily modeled as a group of switches creating a slot signal.

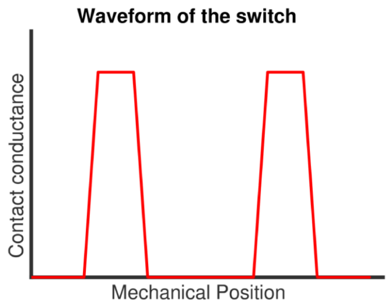

Figure 2. Contact conductance waveform implemented in switch component.

Into the brush-segment switch component, a waveform describing the contact conductance is based on an analytical model. The contact resistance varies from a full contact resistance value to infinity when brush leaves the corresponding segment. So numerically, it is more convenient to use the conductance waveform of Figure 2, to define a switch. Conductance values associated to all switch ensure the current distribution after taking into account flux effects.

This method is an electric component approach and is very common in the literature [5, 6, 7, 9]. It works very well and gives accurate and precise results for slot current waveform. However, one of

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the major drawbacks of this method is that commutator is only described with opening angles and contact resistance. 3D dimensionnal effects are reduced to a simple 1D effect by taking into account both slot and brush width opening and contact resistance. In that way, current flow phenomena are replaced by a circuit approach. Geometries of brushes and segments are not really taken into account and circuit resolution is only an approximation of current flow phenomenon at the electric contact.

2.2. *Projection method applied to commutator*

As previously said, commutator geometry can not be studied in 2D or 3D with the usual approach. Using finite element method to simulate the commutator, instead of circuit component, has been chosen by the authors. The crucial issue about that choice is the modeling of the brush-segment interface to ensure the current distribution. In the corresponding thin meshed region, level-set method is used. A space and time dependent function is defined to describe the conductivity $\sigma(\theta(t))$. The function used in this case is a simply square function from a maximum of contact conductivity (in red on Figure 3) to a minimum of air conductivity (in white). A level-set function is then projected on the mesh of the thin interface between brushes and segments. This function is given by (1), where n_b is the number of brushes, τ_b the brush pitch ratio angle and Θ the mechanical position.

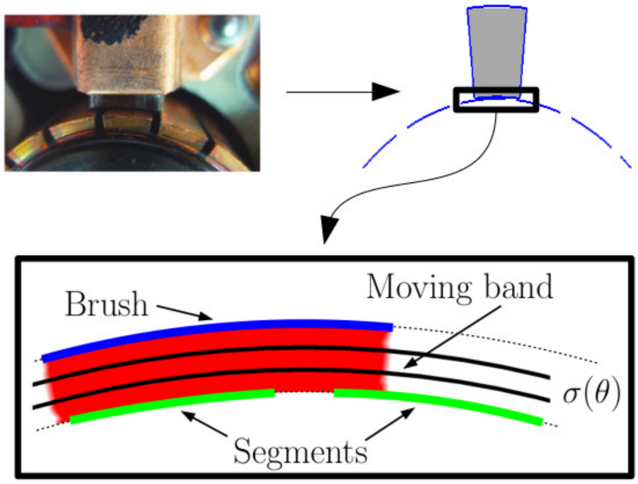

Figure 3. Principle of the proposed commutator model.

$$\sigma(r,\theta) = \begin{cases} \sigma_{\text{copper}}, & \text{if } r \in [R_1, R_2] \text{ and } \\ & (\theta - \Theta) \in [2k\,\tau_b, 2(k+1)\tau_b], \\ & \quad \text{for } k = \{1, \dots, n_b\} \\ \sigma_{air}, & \text{if } r \in [R_1, R_2] \text{ and } \\ & (\theta - \Theta) \notin [2k\,\tau_b, 2(k+1)\tau_b], \\ & \quad \text{for } k = \{1, \dots, n_b\} \\ 0, & \text{if } r \notin [R_1, R_2] \end{cases} \tag{1}$$

with R_1 and R_2 are internal and external radius of commutator interface, and r, θ are cylindro-polar coordinates. Therefore the commutator interface is implemented as a thin meshed region in which a current flow resolution is considered. The movement in this part is taken into account by adding a moving band [10] but in an unusual way. Indeed, it is used with current flow resolution (scalar electric potential v_b) and that is why hybrid projection method is mandatory. Usually, moving band is used in homogenous physical properties and reaffecting the correspondance of nodes does not raise any problems. But at the electric contact, the interface can be seen as a plasma in which conductivity makes the environment inhomogenous. To implement a moving band in this environment allowing a transient current flow resolution, the use of hybrid projection method

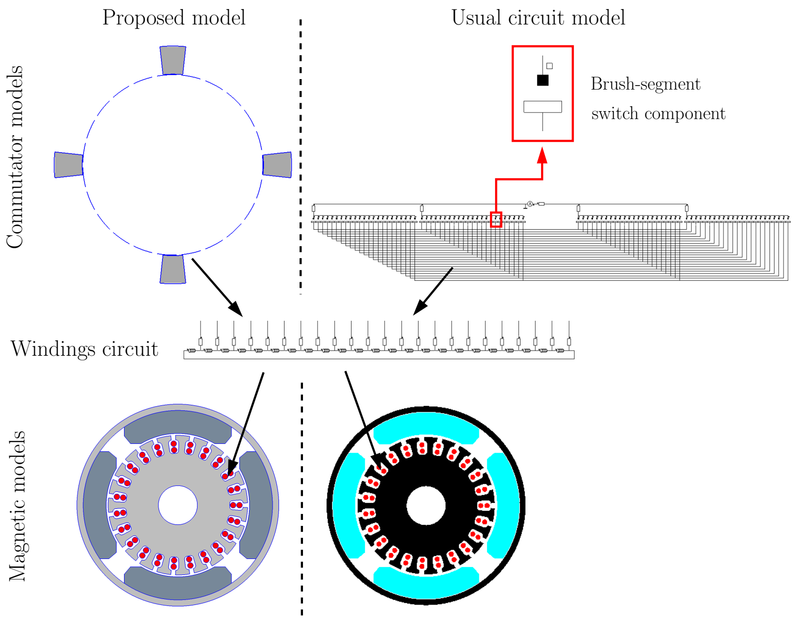

Figure 4. Description of models implementation.

applied on conductivity is required. To sum up, the whole system is modeled using two moving bands: one magnetic and the other one electric. Obviously, usual magnetic moving band and new current flow moving band are linked to follow the same rotation step. Brushes and segments are connected to the other parts of electrical circuit using circuit paths. At the end, the circuit unknowns and the electric potential are strongly coupled through the weak formulation (2), derived from that presented in [11].

Find $v_b, [I_c]$ such that: $\forall v_b' \in H_0^1, \forall [I_c'] \in \mathrm{R}^{n_c}$,

$$\begin{cases} (\sigma(\theta)\,\mathbf{grad}\, v_b, \mathbf{grad}\, v_b')_{\Omega_c} - \langle \mathbf{j}\cdot\mathbf{n}, v_b'\rangle_{\partial\Omega_c} = 0 \\ ([U_c],[I_c'])_{\Omega_{circ}} + ([R]\,[I_c],[I_c'])_{\Omega_{circ}} = 0 \\ \langle \mathbf{j}\cdot\mathbf{n}, v_b'\rangle_{\partial\Omega_c} = \langle I_{c,k}, v_b'\rangle_{\partial\Omega_c}, \forall k \in [1, n_c] \\ \quad (n_c : \text{number of circuit interface connections}) \end{cases} \tag{2}$$

To clearly state about the proposed model, conductivity functions and moving band ensure the slot current distribution. In other words, when a segment is in a contact conductivity area, it allows the current to flow from brushes to corresponding segments. Concerning the choice of the interface domain thickness, brushes-segments contact resistance gives a corresponding surface resistivity calculated with respect to geometric quantities. It means that the interface domain thickness is the choice of the authors provided that conductivity is correctly evaluated giving the expected resistance.

The main advantage of the proposed method is clearly that the commutator system can be studied with its whole geometry. It leads to a full optimization process including DC machines geometry and commutator geometry. Moreover, this proposed model can lead to advanced multiphysics simulations, which may prove useful later on. Indeed, a phenomenon that has been ignored until now is the presence of sparks at the commutation. These occur at switching when the current in the considered section is not equal to the current in the main path. In the particular case of machines with mechanical collectors, some authors model the arc as a constant voltage source [12] as others use static models [13] of the arc in DC applications in which the arc voltage is directly deduced from the current [14]. More accurate spark models are still developed until now and some could be coupled to finite element models [15].

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However, the purpose of this article being to develop a general optimization method for a machine and its power supply, the risk of sparking will not be considered in the following. It is still useful to keep in mind that the armature voltages and currents are available thanks to the circuit model, so a verification of the switching quality may be added to the model in the future.

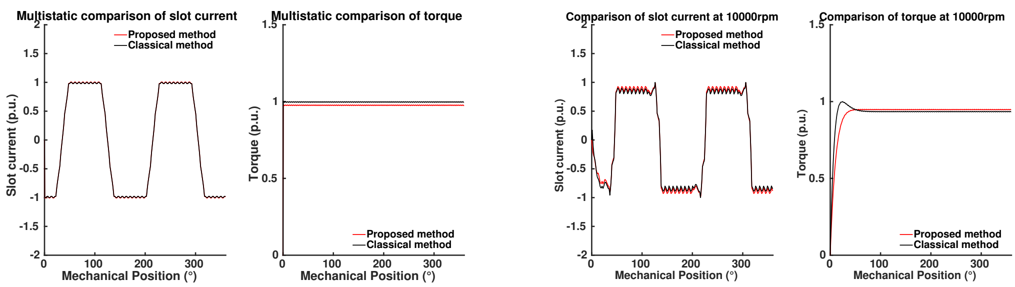
2.3. Comparison of numerical models

An industrial case which considers a DC automotive starter is presented in this part to compare the different approaches used to model a DC commutator. The system is composed of a battery, a usual 4-poles DC motor with ferrite magnets and a commutator to ensure the power supply of the machine through brushes and segments. In order to fairly evaluates both models, the usual method is implemented on commercial FLUX® software. This model will be the reference. The proposed hybrid method is implemented with the ONELAB software bundle. Obviously, same magnetic model and same windings are implemented and meshes are kept as close as possible. The difference only lies in the commutator model and, by the way, the coupling method, as shown in Figure 4.

A transient resolution is implemented on both software and movement is taken into account by moving band method. Accuracy of model is shown thanks to steady-state slot current. In order to save calculation time, it has been decided to implement a speed imposed approach to avoid mechanical transient state. This fact implies a non-significant transient state of current but a conform steady-state. The modeling of the commutator is the cornerstone of the present article, that is why comparison between both approaches relies on slot current waveforms. Nevertheless, electromagnetic torque is also presented in order to observe global electromechanical performances.

In order to simulate a multistatic resolution, a speed of 1 rpm is imposed. In this case, inductive effects can be neglected compared to resistive effects. Results are presented in Figure 5 and are very encouraging. Less than 2% of relative deviation between both models is observed. The resistive commutation is well modeled but it is necessary to observe the inductive commutation.

Taking into account the effect of magnetic flux is mandatory, that's why a speed of 10,000 rpm is imposed. Results are given in Figure 5. At high velocity, for steady-state, less than 5% of relative deviation is observed on slot current and 1% on torque. Moreover, the peak current phenomenon at the beginning of commutation phase is exactly the same with both models. This is an indicator of the possible presence of an electric arc [16].



a) $\Omega = 0$rpm b) $\Omega = 10000$rpm

Figure 5. Method comparison.

Concerning the CPU time, the use of ONELAB software and the implementation of the proposed model give a strong reduction of computation time. Using an Intel Xeon @2.40 GHz with 32 Gb of RAM, the proposed model is almost 7 times faster than the usual one. All the results lead to the conclusion that the commutator is accurately modeled therefore the method can now be used to design a much more sophisticated system.

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3. CLAW POLE MACHINE

3.1. Introduction

The claw pole synchronous machine is a well-known technology for the automotive manufacturer Valeo. It has already been used for years as AC generator for micro and mild hybrid application. This kind of machine is usually characterized by different voltage supplies (12 V to 48 V) and different mechanical power range (up to 6 kW). Obviously, supply is performed by battery associated with an inverter. With the development of small mobility applications, this kind of low power technologies can be used coupled with a reducer to ensure a full electrical powertrain.

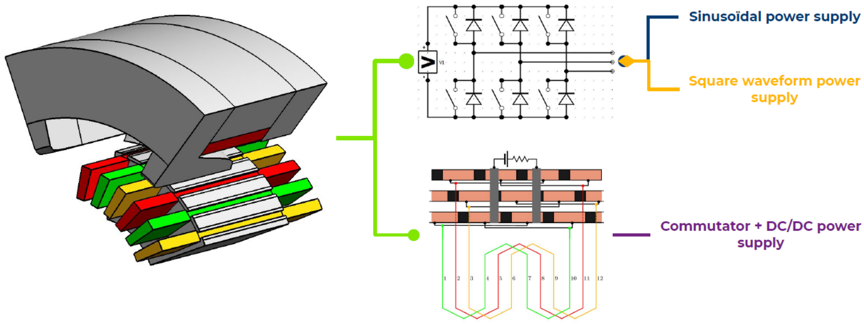

Figure 6. Different supply solution for claw pole machine

An issue in automotive market is to find low-cost solution to electrification. That is why an idea is to replace the electronic inverter by a commutator which, as previously said, can be a robust and reliable signal converter [17]. Both supply solutions can be seen on Figure 6. The purpose will be to develop a numerical model which allows for designing a purely 3D machine with its commutator that respects some structural rules.

3.2. Numerical model

Commutator's brushes are plugged to the supply (battery and DC/DC converter), and segments are connected to winding. On the Figure 7, it can be observed both its schematic representation and its geometric structure in finite element method.

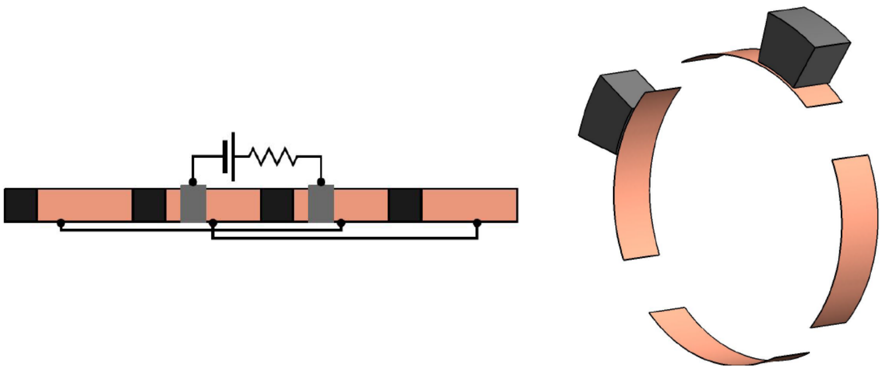

Figure 7. Schematic representation of commutator and its finite element model

Coupled formulations have to be developed in order to connect all parts of the model. Indeed, for the commutator, current flow resolution is implemented to obtain current density but in an other hand, circuit equations are needed to take into account supply and winding. That is why coupling equations are mandatory on brushes and segments to link current and current density. To sum up, circuit and field equations are linked thanks to a strong coupling between:

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- scalar electric potential v_b in the FE domain Ω_c ;
- electrical current I_c in the circuit domain Ω_{circ} ;
- terms of coupling at the interface $\partial\Omega_c$ between both domains.

The weak form associated with this problem is expressed in (3), derived from [11].

$$\begin{cases} \text{Find } v_b \in H_0^1 \text{ and } [I_c] \in \mathbb{R}^{n_b} \text{ such as:} \\ (\sigma(\theta)\,\mathbf{grad}\, v_b, \mathbf{grad}\, v_b')_{\Omega_c} - (\mathbf{j}\cdot\mathbf{n}, v_b')_{\partial\Omega_c} = 0 \\ ([U_c],[I_c'])_{\Omega_{circ}} + ([R][I_c],[I_c'])_{\Omega_{circ}} = 0 \\ (\mathbf{j}\cdot\mathbf{n}, \mathbf{v_b'})_{\partial\Omega_c} = (\mathbf{I_{c,k}}, \mathbf{v_b'})_{\partial\Omega_c}, \forall k \in [1, n_c] \\ \forall v_b' \in H_0^1 \text{ and } \forall [I_c'] \in \mathbb{R}^{n_b} \\ \quad \text{with } n_c \text{ : number of circuit interface connections} \\ \quad \text{with } n_b \text{ : number of circuit branches} \end{cases} \tag{3}$$

The coupling process is graphically represented on Figure 8. Considering δ_{il} the inter-segments

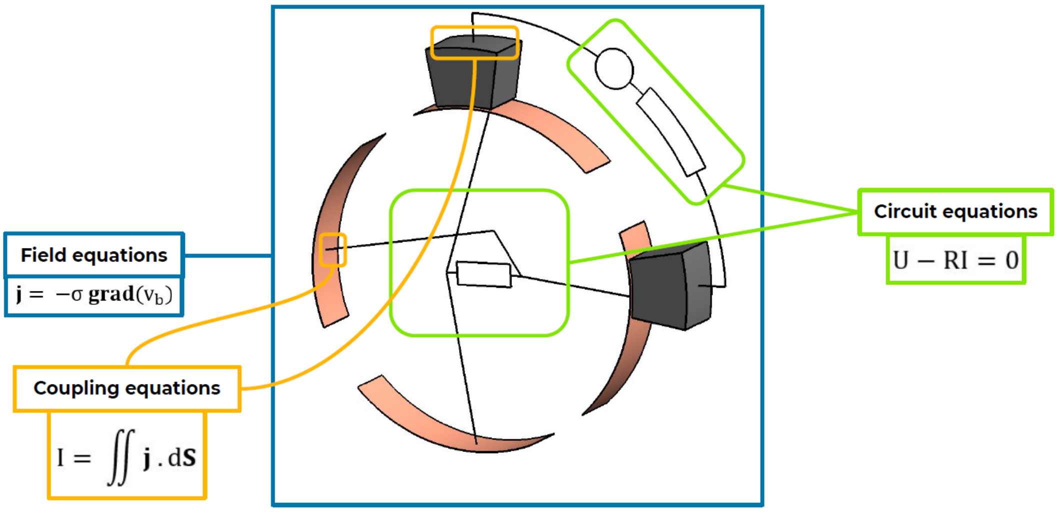

Figure 8. Graphical representation of coupling process for the commutator model

angular opening and δ_b the brush angular opening, two commutator configurations have been arbitrarily chosen as it can be seen in Figure 9 in order to obtain the current waveform in both resistances seen in Figure 8. Two kind of current waveforms can be observed:

- If $\delta_{il} < \delta_b$ then the power supply is short-circuited during an angle equal to $|\delta_b - \delta_{il}|$
- If $\delta_{il} > \delta_b$ then the phase current is null during an angle equal to $|\delta_b - \delta_{il}|$

with δ_b the brush width opening, and δ_{il} the inter-segment width opening. Theoretical results confirm numerical simulation and so the fact that the difference between the angular width of the brush and the inter-segment (i.e the recovery rate) is the key factor which sizing the commutator.

3.3. Design process

Claw-pole machine is used for some small mobilities as the Citroën Ami for example. The aim is to design a commutator respecting the particular specifications of small mobilities. Indeed, the initial DC/AC power supply has to be replaced by a simple DC/DC converter. Due to this change in the power supply line, the initial structure of the claw pole synchronous machine has to be adapted. Claw poles are moved to the stator, and a three rings commutator is plugged to the rotor to supply the machine with a three phase alternative balanced current system, as it can be seen on the Figure 10.

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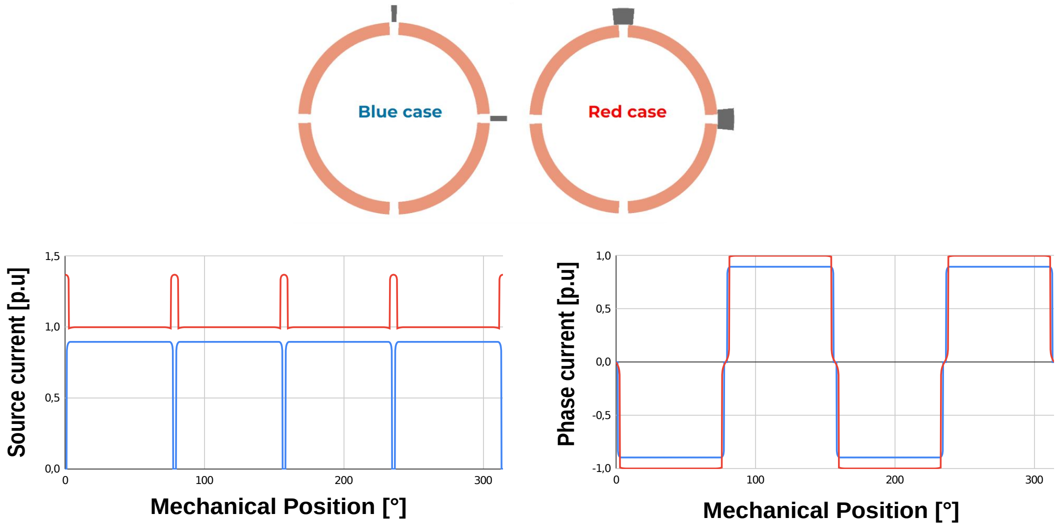

Figure 9. Study cases : δ_{il} fixed to 5.75° and $\delta_b = 2.9^\circ$ (blue case) and $\delta_b = 11.5^\circ$ (red case) and associated current waveform: source current (left) and phase current (right)

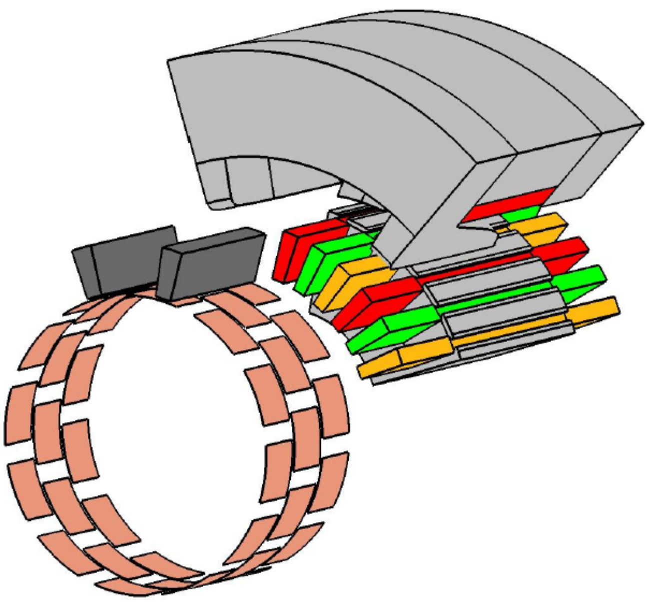

Figure 10. FE-model of claw pole structure with its commutator

Note that in order to reduce CPU time, 2D commutator is implemented in the Onelab software [18, 19]. For same reasons, only static simulations have been performed to obtain the electromagnetic torque. The global magnetostatic problem is divided into 6 coupled formulations:

- 1 current flow formulation to obtain the right power supply with the commutator;
- 4 formulations to compute the source field according to the current in the different coils;
- 1 magnetic scalar potential formulation to obtain global electromagnetic performances.

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To design the machine and its supply simultaneously, the FE model is coupled with a free and open-source optimization framework: NOMAD [20], based on mesh adaptive direct search (MADS) algorithms. The initial geometry is described thanks to 40 real parameters [21] – 2 for the commutator, 7 for the rotor and 31 for the stator. A complex optimization problem consisting in torque maximization under many constraints is then solved:

$$\max_{x \in \mathbb{R}^{40}} (\Gamma_{mean}) \text{ under } \begin{cases} L_B \leq X \\ X \leq U_B \\ \left| \dfrac{\Gamma_{max} - \Gamma_{min}}{\Gamma_{mean}} \right| - 15\% \leq 0 \\ S_{copper_{rotor}} \leq 55\% \ S_{slot} \\ S_{copper_{stator}} \leq 78\% \ S_{slot} \\ J_{stator} \leq 12 \text{ A·mm}^{-2} \end{cases} \tag{4}$$

Convergence of objective function is presented on the Figure 11. It takes around 690 hours (∼1 month) to achieve NOMAD convergence on a Linux workstation (Intel Xeon E5-2637 @3.5 Ghz 48 Go RAM) and around 2 weeks to achieve 93% of the convergence (almost 4 minutes for each blackbox iterations).

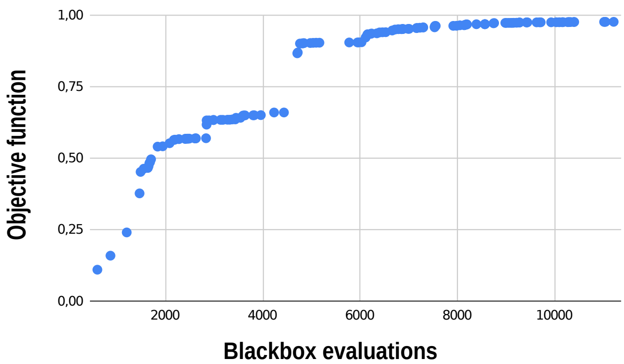

Figure 11. Convergence of objective function vs. number of blackbox evaluations

3.4. *Proof of concept*

The optimization process delivered a result consistent with expectations and compliant with specifications of small mobility applications. That is why a proof of concept of such optimized geometry is realized in order to assess the electrical performances of the machine at low speed.

From left to right, each step of prototyping process is presented on the Figure 12. From optimized geometry, some structural modifications are mandatory. The inclusion of electrical floating segments increase the mechanical stability as the multiplication of brushes which, at the same time, allows for reducing current density. Study is in progress but preliminary results are promising.

Indeed, experimentation shows a deviation of approximately -30% on torque compared to simulation. There may be many reasons for these differences (as switching quality which can be improved [22] as well as brush placement [23]). Nevertheless, some clues show that the global behavior of the machine can be improved by including dynamic simulations straight in optimization process.

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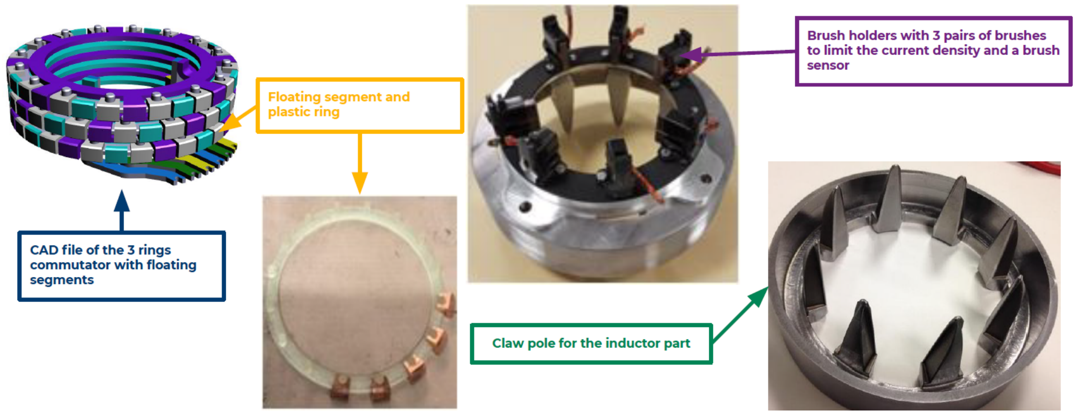

Figure 12. Step of proof of concept development

4. GENERAL CONCLUSION AND PERSPECTIVES

In this paper, a new kind of commutator has been developed. Inspired from usual synchronous machine, design rules have been highlighted in order to adapt DC machines technologies to small mobility applications. There are several advantages to use a mechanical commutator rather than an AC inverter :

- it is a rustic and reliable system mainly due to the reduction of electronic components ;
- it is a competitive low-cost system ;
- the complexication of the machine does not impact the commutator, for example the increase of phase number does not require more transistors but makes it possible to have better performances.

However, this system implies some concessions on the performances, for example the losses will be difficult to evacuate, the acceleration performances will be lower because of the important inertia of the rotor and other mechanical disadvantages of the commutator [24]. Moreover, it generally requires compensation windings and commutation poles to improve the power range.

A coupled model has been developed based on previous work on projection method. It permits to unlock simulation problems in the modeling of the commutator of DC machines which gave rise to the deposit of an international industrial patent [25]. Indeed, the projection of a conductivity function reinforced by an electrical moving band method allows the implementation of current flow resolution at the commutator interface. Thus, an optimization process has been implemented to take into account the machine and its supply.

A proof of concept has been manufactured thanks to the optimization process. It is yet under study but preliminary results are promising. In future work, the purpose will be to develop a dynamic optimization process to take into account electromotive effect.

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