Design of Electrical Rotating Machines by Associating Deterministic Global Optimization Algorithm With Combinatorial Analytical and Numerical Models

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This paper presents a new methodology of design of electrical rotating machines. The methodology is an extension of previous works of the second author. Indeed, associating combinatorial analytical models with exact global optimization algorithms leads to rational solutions of predesign. These solutions need to be validated by a numerical tool (using a finite-element method) before the expansive phase of hand-making a prototype. Such an automatic numerical tool for computing some characteristic values, such as the torque, was previously developed. The idea of this paper is to extend the exact global optimization algorithm by inserting the direct use of this automatic numerical tool. This new methodology makes it possible to solve design problems more rationally. Some numerical examples validate the usefulness of this new approach.

***Index Terms*—Analytical model, deterministic global optimization, finite-element methods, interval branch and bound algorithm, inverse problem of design, numerical model.**

I. Introduction

NOWADAYS, the problem of the design of electrical machines is understood and formulated as an *inverse problem*. The *direct problem* of design can be defined as follows: *From an electromagnetical actuator where the structure, the dimensions, and the composition are known, compute some characteristic values; for example, the flux density, the torque, etc*. The corresponding *inverse problem* of design is: *From the characteristic values given by the schedule of conditions (for example the torque), get the structure, the dimensions, and the composition of the required actuator*. Such a problem is *ill-posed* in the Hadamart sense. Indeed, the existence and the uniqueness of the solution cannot be guaranteed. Furthermore, this problem may generate an infinity of solutions. Many counter-examples can be found, like the machine considered in [14]. The inverse problem of design of electrical machines is explained in [3].

In the classical literature about modeling and optimal design of electrical rotating machines, some particular inverse problems, named predimensioning problems of design, were considered. These works focus on the association of analytical models with local standard optimization algorithms; see [7], [17], [21], and [22]. Thus, first optimal solutions were found for some electrical machines. The solutions obtained by local optimization methods were dependent to the starting point introduced by the user. Consequently, a continuous exact global optimization algorithm based on interval analysis, developed by the second author, was used with efficiency to solve some of these predimensioning problems of design, [14]; stochastic global optimization algorithms are not well adapted to solve this kind of problem because there are too many hard constraints (such as the torque of the machine which is fixed to a value). In [14], it is proved that even if the considered electrical machines are simple (a slotless machine with magnets), the exact solutions were not found by using local search algorithms; see [7] and [22] to compare results with the exact ones published in [14]. For more general inverse problems of design, the second author with Nogarede and Fitan in [2] and [3] considered that the structure and also the components of an electrical machines are variables of the problems, generating *mixed constrained global optimization problems* (including continuous and discrete variables). Therefore, in [2] and [3], a rational way associating *combinatorial analytical models* of electrical rotating machines and an *exact global optimization algorithm*, named IBBA, was proposed and studied. IBBA is an exact global optimization method based on a Branch and Bound technique which uses an interval analysis tool; see [5], [6], and [18] for details on such algorithms and on its rigorous convergence to the global optimum. The obtained solutions satisfy the imposed schedule of conditions via combinatorial analytical models. When these optimal solutions are validated by the means of numerical tools (such as finite-element methods), we denote some differences about the values of the electromagnetic torque. Thus, this involves some adjustments of the parameters of the obtained machines. These adjustments can be done by solving iteratively the direct problem of design until the schedule of conditions will be satisfied using a numerical model. In some numerical tools, such as ANSYS, stochastic and local optimization algorithms can be used to solve this problem of optimal adjustments. We propose in this work to find the exact solution of an optimal design problem which directly satisfies, using numerical tools, the imposed schedule of conditions.

In order to simplify the resolution of the direct problem of design by using a finite-element method, we have proposed a numerical tool, named NUMT, which can automatically mesh, draw, and compute the torque of an electrical machine only defined by its parameters of design, [8]. Thus, the computation of the electromagnetic torque can be performed without the drawing of the motor.

The purpose of this paper is to extend the algorithm IBBA [3], [11] by inserting some steps of NUMT [8] in order to solve, in another more accurate way, the inverse problem of design of electrical rotating machines; see [4] for a preliminary work on this subject. In Section II, we review the rational methodology published in [3] which deals about the association of combinatorial analytical models and IBBA. We then present, in Section III, the numerical tool NUMT [8] which makes it possible to solve automatically the direct problem of design by using a numerical model. Section IV is dedicated to the new methodology combining IBBA and NUMT. In Section V, the new algorithm named IBBA+NUMT is validated on some examples of design of some electrical machines. The solutions are discussed and compared with those produced by IBBA alone.

II. Resolution of the Inverse Problem via Combinatorial Analytical Models and IBBA

The purpose of the paper [3] was to propose a rational methodology for solving the inverse problem of design. Thus, new analytical models, named *combinatorial analytical models*, allowed us to take into account a lot of distinct rotating electrical machines with permanent magnets. These combinatorial analytical models were done by introducing discrete variables into dimensional analytical models; for example the number of pole pairs, the kind of structure (internal or external rotor configuration), and the kind of materials used for the magnets. We then obtain a combinatorial analytical model which represents a large part of electrical rotating machines with magnetic effects; see [2] and [3]. By combining this general combinatorial model with IBBA, which is an efficient exact global optimization code developed by the second author [3], [11], some exact optimal solutions were found by minimizing the magnet volume, the active part volume, the total volume, the mass, or a combination of these criteria with a fixed torque; see [2] and [3]. This methodology is perfect in the first steps of the design of an electrical machine in order to propose solutions which satisfy at best an imposed schedule of conditions.

The inverse problems considered in [3] are formulated as *mixed constrained global optimization problems*:

$$\begin{cases} \min\limits_{\substack{x\in\mathbb{R}^n, z\in\mathbb{N}^m,\\ \sigma\in\prod_{i=1}^{l}K_i, b\in B^r}} & f(x,z,\sigma,b) \\ \text{subjected to} & \\ & g_i(x,z,\sigma,b)\leq 0 \ \forall i\in\{1,\dots,p\} \\ & h_j(x,z,\sigma,b)=0 \ \forall j\in\{1,\dots,q\} \end{cases} \quad (1)$$

where f is a real function, K_i represents an enumerated set of categorical variables, for example the type of magnet, and $B=\{0,1\}$ the boolean set which is used to model the fact that an actuator is with or without slot(s) for example. $\mathbb{R}$ and $\mathbb{N}$ are respectively the real and the positive integer sets. This formulation is called *optimal design optimization problem* and answers perfectly to the *inverse problem* of the design of electromechanical actuators; see [3] and [14] for more details about this formulation.

To solve these problems (1), we must use an exact global optimization algorithm in order to characterize the solution of the problem which can establish that one structure is more efficient than another (with respect of the dimensions). For the use of IBBA, all the functions must be explicitly defined.

A. Algorithm IBBA

Interval analysis was introduced by Moore [16] in order to control the propagation of numerical errors due to floating point computations. Thus, Moore proposes to enclose all real values by an interval where the bounds are the two closest floating point numbers. Then expanding the classical operations—addition, subtraction, multiplication, and division—into intervals, defines interval arithmetic. A straightforward generalization allows computation of reliable bounds (excluding the problem of numerical errors) of a function over a hypercube (or box) defined by an interval vector. Moreover, classical tools of analysis such as Taylor expansions can be used together with interval arithmetic to compute more precise bounds [16]. Other new bounding techniques include combining linear bounds at all vertices of the box [12] or using *affine arithmetic*, [9], [15]. Extensions of these methods are proposed in [3] and [11] in order to solve mixed (discrete and continuous) problems of type (1).

The principle of IBBA is to bisect the initial domain where the solution is sought for into smaller and smaller boxes, and then to eliminate the boxes where the global optimum cannot occur. Elimination of boxes is done by:

- proving, using interval bounds, that no point in a box can produce a better solution than the current best one;
- proving (with interval arithmetic) that at least one constraint cannot be satisfied by any point in such a box.

To accelerate the convergence, constraint propagation techniques are used in some steps of IBBA; see [10] for details. The principle is to use, *a priori*, the implicit relations between the variables which are induced by the constraints in order to reduce the size of a box.

Such interval Branch and Bound algorithms guarantee to produce an ϵ-global optimal solution, where $\epsilon(>0)$ is the maximal error on the objective function value. For details and rigorous convergence analysis of these deterministic global optimization methods based on interval analysis, the reader is invited to consult the three following books: [5], [6], and [18]. For details on IBBA dedicated to solve electromagnetical rotating machines and other actuators, see [3], [11], [13], and [14].

B. Combinatorial Models for Electrical Machines

Hereafter, the analytical equations of the magnetical model are reviewed; see [2] and [3] for details.

The parameters of a rotating electrical machines are as follows: $D(m)$ represents *the bore diameter*, $L(m)$ is *the length*, $l_a(m)$ *the thickness of the permanent magnets*, $E(m)$ *the winding thickness*, $C(m)$ *the thickness of yoke*, β *the polar arc factor*, $g(m)$ *the thickness of the mechanical air gap*, p *the number of pole pairs*, $2m$ *is the number of slots per pole and per phase*, $\mathbf{J}(\sigma_{\mathbf{m}})$ *the magnetic polarization* which depends on the categorical variable $\sigma_{\mathbf{m}}$ representing the type of permanent magnet. Another categorical variable denoted by σ_{mt} defines the type of magnetic conductor, etc., for details; see [2] and [3]. The parameters in bold represent functions depending on the parameters explained above:

$$\begin{aligned}
\mathbf{\Gamma_{em}}(D,L,\ldots) &= \mathbf{k_\Gamma} D\left[D+(1-b_e)(2b_r-1)E\right]L\mathbf{B_e}\mathbf{K_S},\\
\mathbf{K_S}(D,L,\ldots) &= k_r E j\left(b_e\frac{a}{a+d}+(1-b_e)\right),\\
\mathbf{k_\Gamma}(D,L,\ldots) &= \frac{\pi}{2}\left[b_f[1-\mathbf{K_f}]\sqrt{\beta}\right.\\
&\qquad\left.+(1-b_f)\frac{\sqrt{2}}{2}\sin\left(\beta\frac{\pi}{2}\right)\right],\\
\mathbf{K_f}(D,L,\ldots) &= 1.5p\beta\left[\frac{E+g}{D}\right](1-b_e).b_f,\\
\mathbf{B_e}(D,L,\ldots) &= \frac{2\mathbf{J}(\boldsymbol{\sigma}_\mathbf{m})l_a}{(2b_r-1)D\ln\left[\frac{D+2E(2b_r-1)(1-b_e)}{D-2(2b_r-1)[l_a+g]}\right]}\frac{1}{\mathbf{k_c}},\\
\mathbf{k_c}(D,L,\ldots) &= \frac{1}{1-b_e\left[\frac{\mathbf{N_e}a^2}{5\pi D.g+\pi D.a}\right]},\\
\mathbf{N_e}(D,L,\ldots) &= \pi\left[\frac{D+(2b_r-1)E}{d+a}\right],\\
\mathbf{N_e}(D,L,\ldots) &= 2pqm,\\
\mathbf{B_t}(D,L,\ldots) &= \frac{a+d}{d}\mathbf{B_e},\\
\mathbf{B_c}(D,L,\ldots) &= \frac{D}{2pC}\left[\beta\frac{\pi}{2}(1-b_f)+b_f\right]\mathbf{B_e},\\
\mathbf{k_d}(D,L,\ldots) &= b_e\frac{d}{d+a},
\end{aligned}$$

where the generic expression of the electromagnetic torque is denoted by $\mathbf{\Gamma_{em}}$.

$\mathbf{K_S}$ represents the current electric loading. According to the considered kind of armature (nonslotted or slotted), $\mathbf{K_S}$ is identified with two distinct functions. In the case of nonslotted machines, this function is written $\mathbf{K_S} = k_r E j$ whereas for slotted machines $\mathbf{K_S} = k_r E j a/(d+a)$, where j is the current density. A generic formulation of the current electric loading can then be elaborated by introducing a boolean variable (zero or one) b_e. When b_e has value zero, nonslotted machines are considered, and when it is one, slotted machines are taken into account. $\mathbf{k_\Gamma}$ is the torque coefficient, the expression of which depends mainly on the kind of waveform which has been chosen (sinusoidal or rectangular). This coefficient is written $(\pi/2)(1-K_f)\sqrt{\beta}$ for rectangular waveform machines and $(\pi\sqrt{2}/4)\sin(\beta(\pi/2))$ for sinusoidal waveform machines. The elaboration of a generic expression is proposed by introducing a new boolean variable b_f. If b_f is equal to zero, sinusoidal waveform machines are considered and if b_f is equal to one, rectangular waveform machines are taken into account.

Concerning nonslotted machines with rectangular waveform, a semi-empiric magnetic leakage $\mathbf{K_f}$ is proposed. $\mathbf{B_e}$ represents the no-load magnetic radial flux density to the bore diameter neighborhood, which is supposed purely radial in the air gap. An analytical expression for nonslotted machines with internal rotor has already been elaborated; see [14]. A generic formulation of $\mathbf{B_e}$ permits on the one hand to take into account the kind of armature by using the b_e boolean variable, and on the other hand to take into account the rotoric configuration (internal or external) by introducing an extra boolean variable b_r ($b_r = 1$ for an internal configuration and $b_r = 0$ for an external one).

TABLE I
MATERIAL CHARACTERISTICS

Material	Categorical Variable		Kind of materials	J (T)	$\mathbf{B_M}$ (T)	ρ (kg.m^{-3})
Permanent magnet	$\sigma_m =$	1	plastic	0.6	-	$\rho_{PM} = 6000$
		2	NdFeB	0.9	-	$\rho_{PM} = 7900$
Magnetic material	$\sigma_{mt} =$	1	stamping	-	1.5	$\rho_{CM} = 7900$
		2	powder	-	1.2	$\rho_{CM} = 6000$
Electrical conductor	-		copper	-	-	$\rho_{CO} = 8920$

In order to take into account the influence of slots on the distribution of the magnetic induction in the air gap, we use the Carter coefficient $\mathbf{k_c}$, which is equal to 1 for nonslotted machines and is superior to this value for slotted ones. It depends on some geometric parameters and the number of slots $\mathbf{N_e}$. There are two ways for computing this function relating to a geometric relation and an expression of electric origin. These two relations lead to an equality constraint.

$\mathbf{B_t}$ and $\mathbf{B_c}$ are respectively the flux density inside the teeth and the yoke. They are limited to a maximal value $\mathbf{B_M}$ which depends on the kind of magnetic material. Under this value, the material behavior is supposed linear. So we can introduce two inequality constraints linked to the kind of magnetic material, given by

$$\mathbf{B_c} \le \mathbf{B_M}(\sigma_{\mathrm{mt}}) \tag{2}$$

$$b_e\,\mathbf{B_t} \le \mathbf{B_M}(\sigma_{\mathrm{mt}}). \tag{3}$$

Numerical values of $\mathbf{B_M}$ depending on the categorical variable σ_{mt} appear in Table I. $\mathbf{k_d}$ is a specific function added to take into account the shape of the slots. It is equal to the ratio between the tooth width and the width of the tooth pitch. These definitions associating with a schedule of condition define the constraints of the problem. For details on such a model, see [2] and [3].

The other relations define the volumes of the active parts of the machines, the permanent magnet volume $\mathbf{V_m}$, the yoke volume $\mathbf{V_c}$, the teeth or wedge volume $\mathbf{V_d}$, the electrical conductors volume $\mathbf{V_{co}}$, and the global volume $\mathbf{V_g}$, by considering that the slots, wedges, and magnets have a radial geometrical form:

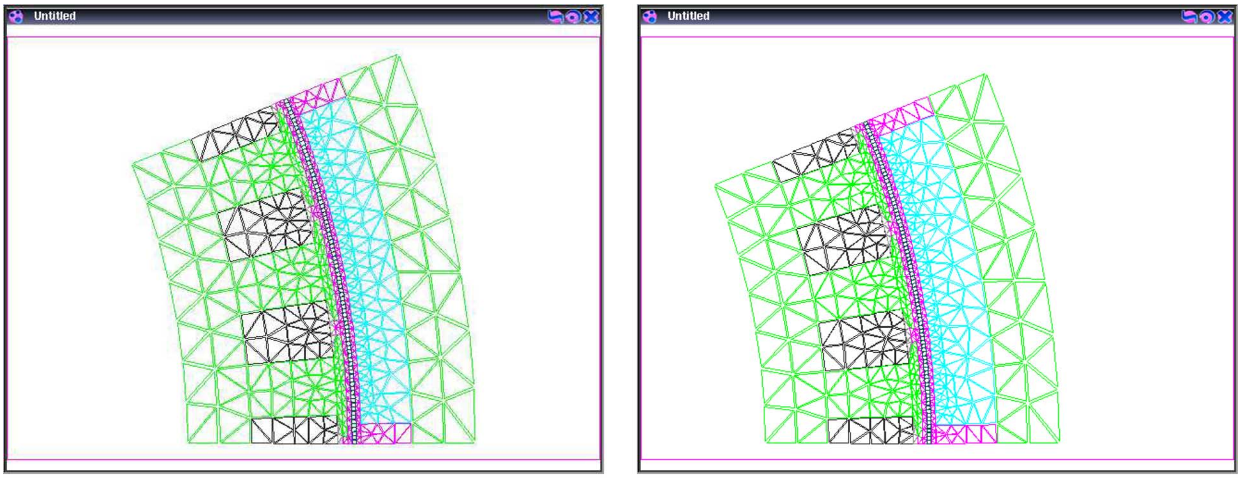
$$\mathbf{V_m}(D,L,b_e,\ldots) = \beta\pi L l_a\left[D-\mathbf{S}(\mathbf{b_r})[2g+l_a]\right] \tag{4}$$

$$\mathbf{V_c}(D,L,b_e,\ldots) = 2\pi LC\left[D+\mathbf{S}(\mathbf{b_r})[E-g-l_a]\right] \tag{5}$$

$$\begin{aligned}\mathbf{V_d}(D,L,b_e,\ldots) &= \pi LE\left[D+\mathbf{S}(\mathbf{b_r})E\right]\\ &\quad\times\left[[1-\beta]b_e+\left[\frac{d}{d+a}\right](1-b_e)\right]\end{aligned} \tag{6}$$

$$\begin{aligned}\mathbf{V_{co}}(D,L,b_e,\ldots) &= k_r\pi LE\left[D+\mathbf{S}(\mathbf{b_r})E\right]\\ &\quad\times\left[\beta.(1-b_e)+\left[\frac{a}{d+a}\right]b_e\right]\end{aligned} \tag{7}$$

$$\begin{aligned}\mathbf{V_g}(D,L,b_e,\ldots) &= \frac{\pi L}{4}\left[b_r\left[D+2[E+C]\right]^2+(1-b_r)\right.\\ &\quad\left.\times\left[D+2[g+l_a+C]\right]^2\right].\end{aligned} \tag{8}$$



(a) (b)

Fig. 1. Meshes of the two solutions corresponding to the minimization of the multicriteria in Table II, displayed in the same zoom box. With the same scale, some differences can be noticed. (a) IBBA. (b) IBBA+NUMT.

A last relation is dedicated to the mass of the active parts $\mathbf{M_a}$:

$$\begin{aligned}\mathbf{M_a}(D, L, b_e, \ldots) &= \mathbf{V_m}.\boldsymbol{\rho}_{\mathbf{PM}}(\boldsymbol{\sigma}_{\mathbf{m}}) + \mathbf{V_c}.\boldsymbol{\rho}_{\mathbf{CM}}(\boldsymbol{\sigma}_{\mathbf{mt}}) \\ &\quad + \mathbf{V_{co}}.\rho_{co} + \mathbf{V_d}\left[\boldsymbol{\rho}_{\mathbf{CM}}(\boldsymbol{\sigma}_{\mathbf{mt}}).b_e + \rho_{Al}.(1 - b_e)\right] \quad (9)\end{aligned}$$

where ρ_{Al}, ρ_{co}, $\rho_{CM}(\sigma_{mt})$, and $\rho_{PM}(\sigma_m)$ are the densities of the aluminum, the copper, the magnetic conductor σ_{mt}, and the permanent magnet σ_m, respectively. Relations linked to volumes and weight will be the objective functions f of the corresponding global optimization problems.

The strong equality constraint is about the torque $\mathbf{\Gamma_{em}}$ which is fixed to a value by the schedule of conditions, denoted in this paper by Γ, i.e., $\mathbf{\Gamma_{em}}(D, L, b_e, \ldots) = \Gamma$. In the following, this constraint is replaced by a numerical computation.

III. Automatic Numerical Tool to Validate Optimal Solutions of Design

Before the phase of prototype making the optimal solutions obtained by the methodology described in [3] need to be validated by using numerical tools, such as finite-element methods EFCAD [1] or ANSYS for example. Some differences between the analytical and numerical values are denoted concerning the electromagnetic torque and then the optimal solution found by the rational methodology proposed in [3] must be adjusted.

An in-depth analysis shows that the problem of comparing analytical and numerical results is a very complicated one. Indeed, the general analytical model is based on some restrictive assumptions which are taken into account in order to develop its equations. This model comes from the electromechanical conversion and the flux conservation by assuming that the magnetic induction in the air gap is purely radial. The respective permeabilities of magnets and iron are fixed as unity and infinity. First, analytical models for nonslotted machines are developed, and then they were extended to slotted machines thanks to the function $\mathbf{K_S}$ (which gives current electric loading, with $b_e = 1$) and to the Carter coefficient $\mathbf{k_c}$; see Section II-B and [2], [3].

The magnetic flux density computation using finite-element methods [19], is more accurate than the analytical one. Nevertheless, we must do some other assumptions. At a design stage, waveforms of the flux and the feeding currents of the electrical machines are assumed to be ideal: rectangular, trapezoidal, or sinusoidal. So the performances or the characteristics of electrical machines can be deduced from flux computations. For instance, for a permanent-magnet machine, the no-load flux in windings due to magnets (Φ_0) and the flux in windings for two types of load currents (longitudinal and transversal, which give the longitudinal and transversal inductances L_d and L_q) are computed. From these three values, the torque, flux, and voltage can be calculated for any type of sinusoidal currents. The electromagnetic torque can be expressed as follows:

$$\mathbf{NUMT}(D, L, \ldots) = 3p\left(\Phi_0 I \cos\psi - \frac{L_d - L_q}{2} I^2 \sin 2\psi\right) \quad (10)$$

where I is the circuit current and ψ is the phase angle difference between the current and the electromotive force.

In order to make the validation phase easier, we developed a numerical tool, named NUMT, [8]. This algorithm is able to translate the values of parameters issued from optimal solutions via IBBA or other ones given by the user. Then, NUMT draws and meshes automatically the corresponding machine. The meshing is performed using simple laws which divided the different regions of drawing in a well adapted way, before calling "Triangle" a free 2-D mesh generator [20]; two examples of obtained meshes are shown in Fig. 1. The flux computations follow and can be performed with or without the drawing of the machine. This tool is very useful to validate our analytical global optima; see [8] for details.

IV. New Methodology of Design

The purpose of this work is to answer the following question: is it possible to replace in (1), the first strong equality constraint $\mathbf{\Gamma_{em}}(\ldots) = \Gamma$ (corresponding to $h_1(\ldots) = 0$) by $\mathbf{NUMT}(\ldots) = \Gamma$?

$$\begin{cases} \min\limits_{\substack{x \in \mathbb{R}^{n_r}, z \in \mathbb{N}^{n_e}, \\ \sigma \in \prod_{i=1}^{n_c} K_i, b \in B^{n_b}}} f(x, z, \sigma, b) \\ g_i(x, z, \sigma, b) \leq 0 \; \forall i \in \{1, \ldots, n_g\} \\ h_j(x, z, \sigma, b) = 0 \; \forall j \in \{2, \ldots, n_h\} \\ \mathbf{NUMT}(x, z, \sigma, b) = \Gamma \end{cases} \quad (11)$$

i.e., find the solution which satisfies the value of the torque by using a finite-element method. Such a constraint $\mathbf{NUMT}(x,z,\sigma,b) = \Gamma$ is named a *black box constraint* because it depends on an algorithm for computing it. Such a new problem (11) is impossible to be solved using IBBA because for using interval analysis tools, all the expressions of the objective or constraint functions must be given explicitly. Indeed, in our knowledge, it is impossible to compute bounds (in a polynomial time) using interval analysis or other tools for the function $\mathbf{NUMT}$ over a box of the initial domain of research. Note that for the equality constraints the index j starts from 2. This is due to the fact that compared to (1) the first equality constraint corresponding to the torque ($\mathbf{\Gamma_{em}}(x,z,\sigma,b) = \Gamma$) is deleted and replaced by the numerical one $\mathbf{NUMT}(x,z,\sigma,b) = \Gamma$.

Therefore, is it possible to consider a more interesting inverse problem of design than (1), which is defined as follows:

$$\begin{cases} \min\limits_{\substack{x\in\mathbb{R}^{n_r},z\in\mathbb{N}^{n_e},\\ \sigma\in\prod_{i=1}^{n_c}K_i,b\in B^{n_b}}} f(x,z,\sigma,b) \\ g_i(x,z,\sigma,b) \le 0 \;\forall i \in \{1,\ldots,n_g\} \\ h_j(x,z,\sigma,b) = 0 \;\forall j \in \{2,\ldots,n_h\} \\ (1-pc)\times\Gamma \le \mathbf{\Gamma_{em}}(x,z,\sigma,b) \le (1+pc')\times\Gamma \\ \mathbf{NUMT}(x,z,\sigma,b) = \Gamma \end{cases} . \quad (12)$$

The purpose of this work is to extend the code IBBA by introducing some steps of NUMT in order to solve problems of type (12) applied to rotating machines with permanent magnets. Indeed, the combinatorial analytical model permits to lead such an algorithm to the determination of the global optima. The idea is to find a solution which satisfies numerically in place of analytically the equality constraint of the torque. The analytical computations of the torque are used to determine the domain where some numerical evaluations must be performed: $(1-pc)\times\Gamma \le \mathbf{\Gamma_{em}}(x,z,\sigma,b) \le (1+pc')\times\Gamma$, where pc and pc' are real values in [0,1[which permit to define the domain of research. Therefore, each optimal solution which is found by using the combination of IBBA and NUMT, named IBBA+NUMT below, satisfies numerically the equality constraint on the fixed torque: $\mathbf{NUMT}(x,z,\sigma,b) = \Gamma$. The obtained solutions are the exact global ones of mathematical program (12); attention must be paid for users of IBBA+NUMT to the definition of pc and pc' (during the following numerical experiments, pc and pc' are fixed to 0.1). In fact, the analytical model is just used to determine a small zone where the numerical solution is sought for, hence if this zone is too reduced then the true numerical optima of problem (11) cannot be reached and if the zone is too large the algorithm could not converge.

In the following, algorithm IBBA+NUMT is detailed in order to solve more general inverse problems of type (12).

Algorithm IBBA+NUMT:

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1) Set $X :=$ the initial domain in which the global minimum is sought for, $X \subseteq \mathbb{R}^{n_r} \times \mathbb{N}^{n_e} \times \prod_{i=1}^{n_c} K_i \times B^{n_b}$.
2) Set $\tilde{f} := +\infty$.
3) Set $\mathcal{L} := (+\infty, X)$.
4) Extract from $\mathcal{L}$ the lowest lower bound.
5) **Bisect the considered box** chosen by its midpoint, yielding V_1, V_2.
6) For j := 1 to 2 do
   a) **Compute** $v_j := lb(f, V_j)$ (**a lower bound of** f **over** V_j).
   b) **Compute all the lower and upper bounds** of all the analytical constraints on V_j; deduction steps using the analytical constraints of (12) permit to reduce V_j, [10].
   c) if $\tilde{f} \ge v_j$ and no analytical constraint of (12) is unsatisfied then
      - insert (v_j, V_j) in $\mathcal{L}$.
      - set m the midpoint of V_j.
      - if m satisfies all the analytical constraints and then if the numerical constraint $\mathbf{NUMT}(x,z,\sigma,b) = \Gamma$ is also satisfied then $\tilde{f} := \min(\tilde{f}, f(m))$.
      - if $\tilde{f}$ is changed then remove from $\mathcal{L}$ all (z, Z) where $z > \tilde{f}$ and set $\tilde{y} := m$.
7) If $\tilde{f} - \min\limits_{(z,Z)\in\mathcal{L}} z < \epsilon$ (where $z = lb(f, Z)$) then STOP.
   Else GoTo Step 4.

We call the *analytical constraints*, all the constraints (including $(1-pc)\times\Gamma \le \mathbf{\Gamma_{em}}(x,z,\sigma,b) \le (1+pc')\times\Gamma$) except the last one $\mathbf{NUMT}(x,z,\sigma,b) = \Gamma$. Because the algorithm stops when the global minimum is sufficiently accurate less than ϵ, one ϵ-global numerical solution is reached: $\tilde{y}$ corresponding to $\tilde{f}$. However, it can be possible that a better solution exists in the sub-boxes remaining in the list $\mathcal{L}$ at the end of the algorithm. Nevertheless, even in the best case, its corresponding minimal value will not be less than $\tilde{f} - \epsilon$. Therefore, by correctly fixing the ϵ value, the obtained solution $(\tilde{y}, \tilde{f})$ is sufficiently useful and can be considered as the global solution (more precisely the ϵ-global solution) of the considered problem (12). For details on IBBA, the way to bisect a box, to compute bounds, to propagate the constraints, to stop the algorithm, etc., see [3], [5], [6], [10], [11], [14], and [18].

The way to correctly define the parameters pc and pc' is not so easy. In this first study, we consider that variations about 10% around the analytical value of the torque are significant enough, i.e., $pc = pc' = 0.1$.

V. Examples of Design

The global optimization algorithms used here, IBBA and IBBA+NUMT, are implemented in Fortran 90/95. All computations were performed on an isolated PC from our laboratory with a 1.8-GHz hard disk drive with 7200 rpm and 512 Mb of RAM.

In order to illustrate the use of our new algorithm IBBA+NUMT, let us consider the following optimal design problems of general electrical slotted rotating three-phase machines with permanent magnets. Thus, in the following b_e will be equal to one. Some parameters such as the diameter D, the length L, and the thickness of the magnets l_a are variables. They are all listed in all tables of result. And some other parameters, such as the current density, the kind of waveform, or the winding fitting factor are fixed for these studies. The kind of material is fixed to a modern NdFeB for problems which results are reported in Tables II and IV. For problems where the results are reported in Tables III and V, the kind of materials can take two different values; see Table I.

TABLE II
Comparisons Between IBBA and IBBA+NUMT Algorithms (With $\sigma_m = 2$ and $\sigma_{mt} = 1$)

Parameters: Name	Bounds	Unit	Min V_g: IBBA	Min V_g: IBBA+NUMT	Min M_a: IBBA	Min M_a: IBBA+NUMT	Min $Multi$: IBBA	Min $Multi$: IBBA+NUMT
D	$[0.01, 0.3]$	m	**0.1331**	**0.1310**	0.1400	0.1400	**0.1400**	**0.1310**
L	$[0.01, 0.3]$	m	**0.0474**	**0.0497**	**0.0496**	**0.0519**	**0.0451**	**0.0497**
l_a	$[0.003, 0.01]$	m	0.0047	0.0047	0.0039	0.0039	**0.0039**	**0.0047**
E	$[0.005, 0.03]$	m	0.0074	0.0074	**0.0074**	**0.0075**	**0.0074**	**0.0073**
C	$[0.003, 0.02]$	m	0.0049	0.0049	0.0039	0.0039	**0.0050**	**0.0049**
β	$[0.7, 0.9]$		0.89	0.89	**0.74**	**0.71**	0.89	0.89
k_d	$[0.4, 0.6]$		0.5043	0.5043	**0.4978**	**0.5022**	**0.5043**	**0.4957**
p	$[\![3, 10]\!]$		8	8	8	8	8	8
m	$\{1, 2\}$		1	1	1	1	1	1
b_r	$\{0, 1\}$		0	0	0	0	0	0
Volume		m^3	**$8.881\ 10^{-4}$**	**$9.072\ 10^{-4}$**	$9.716\ 10^{-4}$	$10.157\ 10^{-4}$	$9.067\ 10^{-4}$	$9.072\ 10^{-4}$
Mass		kg	3.21	3.31	**2.94**	**3.07**	3.10	3.30
Multi			2.09	2.15	2.10	2.19	**2.07**	**2.14**
Analytical Torque		N·m	**9.81**	10.00	**9.82**	10.21	**9.86**	9.93
Numerical Torque		N·m	9.35	**9.96**	9.26	**9.86**	9.06	**9.96**
CPU - Time		min	0min35s	7min15s	1min14s	8min17s	1min03s	7min37s
Numerical Computations			-	437	-	560	-	526

TABLE III
Same Tests, Including Materials (and Same $Multi$)

Parameters: Name	Bounds	Unit	Min V_g: IBBA	Min V_g: IBBA+NUMT	Min M_a: IBBA	Min M_a: IBBA+NUMT	Min $Multi$: IBBA	Min $Multi$: IBBA+NUMT
D	$[0.01, 0.3]$	m	**0.1330**	**0.1310**	0.1400	0.1400	0.1350	0.1350
L	$[0.01, 0.3]$	m	**0.0474**	**0.0497**	**0.0497**	**0.0519**	**0.0474**	**0.0497**
l_a	$[0.003, 0.01]$	m	0.0047	0.0047	0.0039	0.0039	0.0044	0.0044
E	$[0.005, 0.03]$	m	0.0074	0.0074	0.0073	0.0073	0.0074	0.0074
C	$[0.003, 0.02]$	m	0.0049	0.0049	0.0050	0.0050	0.0060	0.0060
β	$[0.7, 0.9]$		0.89	0.89	**0.74**	**0.71**	**0.89**	**0.84**
k_d	$[0.4, 0.6]$		0.5043	0.5043	0.4958	0.4958	0.5021	0.5021
p	$[\![3, 10]\!]$		8	8	8	8	8	8
m	$\{1, 2\}$		1	1	1	1	1	1
b_r	$\{0, 1\}$		0	0	0	0	0	0
σ_m	$\{1, 2\}$		2	2	2	2	2	2
σ_{mt}	$\{1, 2\}$		1	1	2	2	2	2
Volume		m^3	**$8.881\ 10^{-4}$**	**$9.072\ 10^{-4}$**	$9.98\ 10^{-4}$	$10.422\ 10^{-4}$	$9.276\ 10^{-4}$	$9.726\ 10^{-4}$
Mass		kg	3.21	3.31	**2.74**	**2.84**	2.93	3.04
Multi			2.09	2.15	2.06	2.14	**2.04**	**2.13**
Analytical Torque		N·m	**9.81**	10.00	**9.82**	10.07	**9.92**	10.10
Numerical Torque		N·m	9.35	**9.87**	9.52	**9.89**	9.30	**9.93**
CPU - Time		min	9min17s	23min49s	23min32s	39min08s	18min15s	42min15s
Numerical Computations			-	466	-	268	-	721

TABLE IV
Same Tests As Table II (With $\sigma_m = 2$ and $\sigma_{mt} = 1$) With $b_r = 1$

Parameters: Name	Bounds	Unit	Min V_g: IBBA	Min V_g: IBBA+NUMT	Min M_a: IBBA	Min M_a: IBBA+NUMT	Min $Multi$: IBBA	Min $Multi$: IBBA+NUMT
D	$[0.01, 0.3]$	m	**0.1313**	**0.1315**	0.1339	0.1339	**0.1339**	**0.1313**
L	$[0.01, 0.3]$	m	**0.0485**	**0.0519**	**0.0496**	**0.0542**	**0.0474**	**0.0530**
l_a	$[0.003, 0.01]$	m	0.0083	0.0083	0.0038	0.0038	**0.0056**	**0.0047**
E	$[0.005, 0.03]$	m	**0.0082**	**0.0081**	0.0082	0.0082	0.0082	0.0082
C	$[0.003, 0.02]$	m	**0.0043**	**0.0050**	0.0043	0.0043	0.0043	0.0043
β	$[0.7, 0.9]$		**0.88**	**0.86**	**0.89**	**0.79**	**0.89**	**0.88**
k_d	$[0.4, 0.6]$		**0.5029**	**0.4958**	0.5028	0.5028	**0.5028**	**0.5029**
p	$[\![3, 10]\!]$		9	9	9	9	9	9
m	$\{1, 2\}$		1	1	1	1	1	1
Volume		m^3	**$9.309\ 10^{-4}$**	**$10.125\ 10^{-4}$**	$9.841\ 10^{-4}$	$10.743\ 10^{-4}$	$9.395\ 10^{-4}$	$10.182\ 10^{-4}$
Mass		kg	3.64	4.09	**3.27**	**3.50**	3.34	3.54
Multi			2.11	2.34	2.06	2.22	**2.03**	**2.18**
Analytical Torque		N·m	**9.97**	10.56	**9.83**	10.10	**9.90**	10.40
Numerical Torque		N·m	8.79	**10.11**	8.99	**9.91**	8.61	**10.04**
CPU - Time		min	0min12s	37min17s	0min17s	24min59s	0min16s	31min25s
Numerical Computations			-	789	-	533	-	677

TABLE V
SAME TESTS AS TABLE IV ($b_r = 1$), INCLUDING MATERIALS

Parameters Name	Bounds	Unit	Min V_g IBBA	Min V_g IBBA+NUMT	Min M_a IBBA	Min M_a IBBA+NUMT	Min $Multi$ IBBA	Min $Multi$ IBBA+NUMT
D	$[0.01, 0.3]$	m	**0.1313**	**0.1315**	0.1339	0.1339	0.1339	0.1339
L	$[0.01, 0.3]$	m	**0.0485**	**0.0519**	**0.0519**	**0.0542**	**0.0519**	**0.0530**
l_a	$[0.003, 0.01]$	m	0.0083	0.0083	0.0038	0.0038	**0.0038**	**0.0047**
E	$[0.005, 0.03]$	m	**0.0082**	**0.0081**	0.0082	0.0082	0.0082	0.0082
C	$[0.003, 0.02]$	m	**0.0043**	**0.0050**	0.0043	0.0043	0.0043	0.0043
β	$[0.7, 0.9]$		**0.88**	**0.84**	**0.81**	**0.79**	**0.81**	**0.88**
k_d	$[0.4, 0.6]$		**0.5029**	**0.4958**	0.5028	0.5028	0.5028	0.5028
p	$[\![3, 10]\!]$		9	9	9	9	9	9
m	$\{1, 2\}$		1	1	1	1	1	1
σ_m	$\{1, 2\}$		2	2	2	2	2	2
σ_{mt}	$\{1, 2\}$		1	1	2	2	2	2
Volume		m^3	**$9.309\ 10^{-4}$**	**$10.125\ 10^{-4}$**	$10.287\ 10^{-4}$	$10.743\ 10^{-4}$	$10.287\ 10^{-4}$	$10.515\ 10^{-4}$
Mass		kg	3.64	4.05	**2.83**	**2.93**	2.83	2.98
Multi			2.29	2.52	2.11	2.19	**2.10**	**2.18**
Analytical Torque		N·m	**9.97**	10.40	**9.83**	10.10	**9.83**	10.22
Numerical Torque		N·m	8.79	**9.95**	9.32	**9.91**	9.32	**9.81**
CPU - Time		min	1min26s	56min00s	1min52s	15min55s	1min32s	31min01s
Numerical Computations			-	1161	-	268	-	531

Three optimization problems are solved—the first two deal with the minimization of a single criterion which are the global volume ($\mathbf{V_g}$) and the mass ($\mathbf{M_a}$), and the third problem is a multicriteria one ($\mathbf{Multi}$):

$$\mathbf{Multi}(\cdots) = \frac{\mathbf{M_a}}{\min_{\mathbf{\Gamma_{em}}=\Gamma} \mathbf{M_a}} + \frac{\mathbf{V_g}}{\min_{\mathbf{\Gamma_{em}}=\Gamma} \mathbf{V_g}} \tag{13}$$

where the weight factors are the inverse of the global optima values of the global volume and mass of problem (1) which are obtained using IBBA; they are denoted by $\min_{\mathbf{\Gamma_{em}}=\Gamma} \mathbf{V_g}$ and $\min_{\mathbf{\Gamma_{em}}=\Gamma} \mathbf{M_a}$. In order to make some comparisons, we use exactly the same criterion for solving the more general associated problem of type (12) by using IBBA+NUMT.

The mechanical air gap is also fixed to the value of 1 mm; if this value were free then it always decreases to its lower bound when we minimize some volumes or the mass. The magnets are a modern NdFeB and the magnetic circuits (yoke and teeth) are made with core of laminated sheets.

The main equality constraint is true when the torque is equal to 10 (+ or -0.2) $\mathrm{N \cdot m}$. For IBBA+NUMT, the zone defined using the analytical model is between 9 and 11 $\mathrm{N \cdot m}$, i.e., $pc = pc' = 0.1$. The stopping criteria in step 7) of Algorithms IBBA+NUMT and IBBA are fixed to $\epsilon = 10^{-7}, 10^{-3}$, and 10^{-3} for V_g, M_a, and $Multi$, respectively. The numerical results presented in Tables II–V correspond to the solving of problem (1) in the columns named IBBA, and of problem (12) in the columns named IBBA+NUMT, respectively.

Even if all the solutions have the same structure (eight pole pairs and an inverse rotoric configuration), the behavior of our new algorithm is not the same in the three cases. First, for the three minimizations, we note that the constraint upon the torque is never satisfied numerically for the solutions obtained by IBBA. Considering the minimization of V_g in Table II, only the two geometrical parameters (D and L) change to reach a correct numerical value for the torque. An expert of the domain would be able to perform these adjustments or equivalent ones. For the minimization of M_a, four parameters (linked to the active parts) change. Comparing IBBA and IBBA+NUMT methods in Table II, the values of the corresponding torque increase about 6.5% in these two first cases, the volume only about 2.2%, and the mass about 4.4%. For the multicriteria minimization in Table II, we note that the numerical torque of the analytical solution is close to 9 $\mathrm{N \cdot m}$. The changes of some parameters of design (for example, D and L) are not so obvious as those for the minimization of the global volume. In this case, IBBA+NUMT found a new optimum which differs from the analytical one for almost all the geometrical parameters. Moreover, we note that the value of the multicriteria increases about 3.4% between IBBA and IBBA+NUMT and all the variables are distinct. Hence, for an expert, this solution (or an equivalent one) should be very difficult to obtain. This last case shows the real efficiency of this new methodology of design perfectly. Furthermore, in Table II, we notice that the values of the criterion of the optima obtained using the IBBA methods are better than those found using IBBA+NUMT. However, the numerical optimal value of the torque for a solution obtained using the IBBA method does not satisfy numerically the main constraint of the torque. Therefore, the analytical optimal solutions of design are less interesting than the numerical ones. We can also note in Table II that the analytical values are very close to the numerical ones for the solutions obtained by using IBBA+NUMT algorithm.

Moreover, for this first example presented in Table II, the computational times are rather short, less than 9 min for the slowest problem which needs 560 runs of NUMT.

Remark 1: Fig. 1 represents the two solutions obtained by IBBA and IBBA+NUMT for the minimization of the criteria Multi of the example presented in Table II. We can note in Fig. 1 that the external radius of the machine is smaller in the case of IBBA+NUMT than with IBBA. Thus, even if the solution of the multicriteria problem obtained by our new algorithm is greater for V_g or $Mass$ compared to the one issued from IBBA, this solution is more efficient with regard to the external volume.

In Table III, we solve the same problem but considering that the kind of materials of the magnet and the iron yoke can have two possible values referenced in Table I. In Table III, all the solutions change between IBBA and IBBA+NUMT depending on the optimization problem.

First, the kind of permanent magnet keeps the same value ($\sigma_m = 1$) in the two problems; see Tables II and III. Indeed, in all case, it is more interesting to use modern magnets rather than plastic ones. For a criterion like V_g, it is totally natural because the density does not intervene in the volume calculation. So the material with the highest magnetic polarization appears in our results. It is not so obvious with criteria like M_a or $Multi$, but we can see that the difference between the densities of the two kind of magnets are not significant enough to compensate the gap between the polarization values. Second, the values of σ_{mt} have changed compared to Table III for the mass and the multicriteria. We obtain better results with powder than with laminated sheets, due to the smaller value of its density. For the global volume minimization, it is natural to obtain stamping again. This one is the material with the highest maximal value of the magnetic flux density, so the iron volume can be smaller than the volume of powder, without reaching its saturation.

We note that the computational times are most important than those presented in Table II where the kind of materials were fixed.

Remark 2: In light of such results, we can think that the kind of magnetic material σ_{mt} may be useless during the resolution of our problems. Indeed, we obtain exactly the same result in including materials or not for the volume minimization and, for the mass minimization it is the less heavy material which appears in our results. Nevertheless, the problems corresponding to the minimization of the global volume have been also computed with a value of σ_{mt} fixed to 2. The obtained results differ from those presented in Tables II and III: $V_g = 9.276 \cdot 10^{-4}$ for IBBA and $V_g = 9.726 \cdot 10^{-4}$ for IBBA+NUMT. These results are quite different (and worse) than the previous ones. It means that the inequality constraints on the maximal value of the flux density in the yoke and the teeth (2), (3) in which appear σ_{mt} has an implicit effect on the optimization problems of design (1) and (12). This remark is also available for the following problems presented in Tables IV and V.

Because all the solutions obtained in Tables II and III are about machines with an external rotoric configuration ($b_r = 0$), we fixed all rotoric configuration to the internal case ($b_r = 1$). The obtained solutions are reported in Tables IV and V, respectively, with or without the fact that the kind of material is fixed or not, as above.

We can note that the value of pole pairs has changed to nine, and all results again have the same value. Different results are obtained comparing the two algorithms IBBA and IBBA+NUMT. Sometimes the differences take into account a lot of parameters (see $\min \mathbf{V_g}$ in the two tables). Furthermore, the solution for the minimization of the mass is more quickly obtained when the parameters of materials are free (see Tables IV and V). This is due to the fact that fixing materials in the third problem involves a more difficult global optimization problem than the more general one including the kind of materials as free parameters.

We note that for all these numerical tests the magnetical torques calculated with NUMT from the solutions given by IBBA are lower than the imposed value fixed to 10 Nm; the lowest obtained value is about 8.61 $\mathrm{N \cdot m}$, see Table IV for the criterion $\mathbf{Multi}$. Nevertheless, the values of the magnetical torque computed with the analytical equation from solutions given by IBBA+NUMT are close to 10 (just above). This point leads to prove that solutions obtained using IBBA+NUMT are more acceptable for a future construction of the machine.

Remark 3: In all the numerical tests presented in this paper, the structure and the composition of the machine are the same comparing solutions obtained with IBBA and IBBA+NUMT; only the continuous parameters of the machine have to be changed. This fact validates the predesigning phase of the use of IBBA and also analytical models.

Remark 4: Using only IBBA and analytical models, all the numerical tests show that the solutions are under the given torque (10 $\mathrm{N \cdot m}$). This seems to show that, when optimal solutions were computed using IBBA and analytical models, this method also maximizes the error (due to the analytical model) about the so-computed torque.

VI. Conclusion

In this paper, we solve for the first time a much more general and useful inverse problem of design [formulated by (12)] than those already solved in [2] and [3]. The obtained solutions using our new algorithm IBBA+NUMT now satisfy the constraint about the torque (which is fixed by the schedule of conditions) in a numerical way. Thus, the generated solutions are directly validated numerically. Of course, problems of type (12) are much more difficult to solve than its corresponding problem considering the analytical equation in place of the numerical constraint (1). Indeed, the examples show the usefulness of the IBBA+NUMT in place of IBBA alone; a lot of continuous parameters can change between the two obtained solutions. In the field of global optimization, this is, in our knowledge, the first time that problems with a black-box constraint are solved by an exact interval Branch and Bound algorithm.

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